Consider the expression $(5x^7 - 11x^3)^{41}$.   2 EACH EXCEPT AS NOTED   SCORE: _____ / 25 PTS

[a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor $_nC_r$ (or equivalent) notation.

$$(5x^7)^{41} + \binom{41}{1}(5x^7)^{40}(-11x^3) + \binom{41}{2}(5x^7)^{39}(-11x^3)^2$$

1

$$= 5^{41}x^{287} - 41\cdot 5^{40}\cdot 11x^{283} + \boxed{\frac{41!}{2!39!}}\,5^{39}\,11^2\,x^{279} \qquad \boxed{\frac{41\cdot \cancel{40}^{20}\cdot 39!}{\cancel{2}\cdot 1\cdot 39!}}$$

$$= 5^{41}x^{287} - 41\cdot 5^{40}\cdot 11x^{283} + 41\cdot 20\cdot 5^{39}\cdot 11^2 x^{279}$$

1   1 $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$

[b] Find the coefficient of x^{211} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT $_nC_r$ (or equivalent) notation.

$$\binom{41}{r}(5x^7)^{41-r}(-11x^3)^r = \binom{41}{r}5^{41-r}(-11)^r x^{7(41-r)+3r}$$

$7(41-r)+3r=211$

$287-4r=211$

$-4r=-76$

$r=19$

$$\text{COEFFICIENT} = \binom{41}{19}5^{22}(-11)^{19} = -\frac{41!}{19!22!}5^{22}11^{19}$$

$1\frac{1}{2}$

Prove that $\sum_{i=1}^{n}(6i+5)4^{i-1}=(2n+1)4^n-1$ for all positive integers n **using mathematical induction**. SCORE: _____ / 25 PTS

2 EACH EXCEPT AS NOTED

BASIS CASE: $\underbrace{\sum_{i=1}^{1}(6i+5)4^{i-1}}=\underbrace{11\cdot 4^0=11=3\cdot 4^1-1}$

INDUCTIVE STEP: ASSUME $\underbrace{\sum_{i=1}^{k}(6i+5)4^{i-1}=(2k+1)4^k-1}$ FOR SOME ARBITRARY INTEGER $k\geq 1$

$$\underbrace{\sum_{i=1}^{k+1}(6i+5)4^{i-1}}$$

$$=\underbrace{\sum_{i=1}^{k}(6i+5)4^{i-1}+(6k+11)4^k}$$

$$=\underbrace{(2k+1)4^k-1}+(6k+11)4^k$$

$$=\underbrace{(2k+1+6k+11)}4^k-1$$

$$=\underbrace{(8k+12)}4^k-1$$

$$=\underbrace{4(2k+3)}4^k-1=\underbrace{(2(k+1)+1)4^{k+1}-1}$$

SO, BY MI,

$$\sum_{i=1}^{n}(6i+5)4^{i-1}=(2n+1)4^n-1$$

FOR ALL POSITIVE INTEGERS n

Find the value of $\sum_{n=2}^{\infty} 700(0.5)^{3n-5}$. **HINT: Write out the first few terms first.**

SCORE: _____ / 15 PTS

$$= 700(0.5) + 700(0.5)^4 + 700(0.5)^7 + \dots$$

INFINITE GEOMETRIC SERIES

$$r = (0.5)^3 = \frac{1}{8}$$

3

$$= \frac{700(\frac{1}{2})}{1 - \frac{1}{8}}$$

4

4

$$= \frac{350}{\frac{7}{8}} = \cancel{350}^{\,50} \cdot \frac{8}{\cancel{7}} = 400$$

4

Consider the sequence defined recursively by $a_n = na_{n-1} + \frac{3}{2}n^2 - 3, \quad a_1 = -\frac{9}{2}$.   **SCORE: _____ / 20 PTS**

[a] Find the first 6 terms of the sequence. **Your answers must be integers or fractions, NOT decimal approximations.**

$a_2 = 2a_1 + \frac{3}{2}\cdot 2^2 - 3 = 2(-\frac{9}{2}) + 6 - 3 = -6$ 3

$a_1 = -\frac{9}{2}$

$a_3 = 3a_2 + \frac{3}{2}\cdot 3^2 - 3 = 3(-6) + \frac{27}{2} - 3 = \frac{-15}{2}$ 3

$a_4 = 4a_3 + \frac{3}{2}\cdot 4^2 - 3 = 4(-\frac{15}{2}) + 24 - 3 = -9$ 3

$a_5 = 5a_4 + \frac{3}{2}\cdot 5^2 - 3 = 5(-9) + \frac{75}{2} - 3 = -\frac{21}{2}$ 3

$a_6 = 6a_5 + \frac{3}{2}\cdot 6^2 - 3 = 6(-\frac{21}{2}) + 54 - 3 = -12$ 3

[b] Based on the first 6 terms, does the sequence appear to be arithmetic, geometric or neither ? Show how you reached your conclusion.

ARITHMETIC 2

$-6 - -\frac{9}{2} = -\frac{3}{2} = d$

$-6 + -\frac{3}{2} = -\frac{15}{2}$

$-\frac{15}{2} + -\frac{3}{2} = -9$

$-9 + -\frac{3}{2} = -\frac{21}{2}$

$-\frac{21}{2} + -\frac{3}{2} = -12$

3

QJ got a new credit card on December 1, 2015, and charged \$47 on it that day. On the 1st day of every month after that, QJ charged \$13 more than he had charged on the 1st day of the previous month. By December 1, 2017, how much had QJ charged on his card altogether ? (Assume that QJ never charged anything else to his card except on the 1st day of each month.)

SCORE: _____ **/ 15 PTS**

$47+(47+13)+(47+13+13)+\cdots+(47+13\cdot(25-1))$ ARITHMETIC SERIES

3

$=\frac{25}{2}(47+47+13(25-1))$ 9

$= 5075$ DOLLARS

3

Simplify $\binom{2n-1}{2n-4}$.

SCORE: _____ / 15 PTS

$$= \boxed{\frac{(2n-1)!}{(2n-4)!\,3!}} = \boxed{\frac{(2n-1)\overset{n-1}{\cancel{(2n-2)}}(2n-3)\cancel{(2n-4)!}}{\cancel{(2n-4)!}\,3\cdot\cancel{2}\cdot 1}}$$

5 5

$$= \frac{1}{3}(2n-1)(n-1)(2n-3)$$

5

Find a_n for the geometric sequence with $a_3 = 54x^4y$ and $a_6 = -\frac{16x^{19}}{y^5}$. **SCORE: _____ / 20 PTS**

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = 54x^4y$$

$$a_6 = a_1 r^5 = -\frac{16x^{19}}{y^5}$$

$$\underbrace{\frac{a_1 r^5}{a_1 r^2} = \frac{\frac{-16x^{19}}{y^5}}{54x^4y}}_{3}$$

$$\underbrace{r^3 = -\frac{8}{27}\frac{x^{15}}{y^6}}_{4}$$

$$\underbrace{r = -\frac{2}{3}\frac{x^5}{y^2}}_{3}$$

$$\underbrace{a_1\left(-\frac{2}{3}\frac{x^5}{y^2}\right)^2 = 54x^4y}_{3}$$

$$a_1 = 54x^4y\left(-\frac{3}{2}\frac{y^2}{x^5}\right)^2$$

$$= \cancel{54}^{27}x^4y \cdot \frac{9}{\cancel{4}_2}\frac{y^4}{x^{10}}$$

$$\underbrace{= \frac{243}{2}\frac{y^3}{x^6}}_{4}$$

$$\underbrace{a_n = \frac{243y^3}{2x^6}\left(-\frac{2x^5}{3y^2}\right)^{n-1}}_{3}$$

Use sigma notation to write the series $-4+25-64+121-\cdots-2500$.

SCORE: _____ / 15 PTS

$-2^2+5^2-8^2+11^2-\cdots-50^2$

$2, 5, 8, 11, \ldots, 50$

ARITHMETIC SEQUENCE

$\underbrace{a_n = 2+3(n-1)=50}_{2}$

$3(n-1)=48$

$n-1=16$

$\underbrace{n=17}_{2}$

$3 \quad \boxed{\sum_{n=1}^{17} \underbrace{(-1)^n}_{2} \underbrace{(2+3(n-1))}_{3}^{\overbrace{2}^{3}}}$

$= \sum_{n=1}^{17} (-1)^n (3n-1)^2$